Large-N behavior of the Wilson loops of generalized two-dimensional Yang-Mills theories

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Abstract

The large-N limit of the expectation values of the Wilson loops in the fundamental representation, corresponding to two-dimensional $\mathrm{U}(N)$ Yang-Mills and generalized Yang-Mills theories on a sphere are studied. The behavior of the expectation values of the Wilson loops both near the critical area and for large areas are investigated. It is shown that the expectation values of the Wilson loops at large areas behave exponentially with respect to the area of the smaller region the boundary of which is the loop; and for the so called typical theories, the expectation values of the Wilson loops exhibit a discontinuity in their second derivative (with respect to the area) at the critical area.

1 Introduction

Two-dimensional Yang-Mills theory (YM_2) and generalized Yang-Mills theories $(gYM_2$'s) have been a subject of extensive study during recent years [1–17]. These are important integrable models which can shed light on some basic features of QCD₄. Also, there exists an equivalence between YM₂ and the string theory. It was shown that the coefficients of the 1/N expansion of the partition function of SU(N) YM₂ are determined by a sum over maps from a two-dimensional surface onto the two-dimensional target space.

The partition function and the expectation values of the Wilson loops of YM_2 have been calculated in lattice- [1,18] and continuum-formulations [4,12,13]. The partition function and the expectation values of the Wilson loops of gYM_2 's have been calculated in [10,11]. All of these quantities are described as summations over the irreducible representations of the corresponding gauge group. In general, it is not possible to perform these summations explicitly. For large gauge groups, however, these summations may be dominated by some specific

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representations, and it can be possible to perform the summations explicitly. There are other physical reasons as well (for example the relation between large- $N \ YM_2$ and the string theory), that make the study of the YM_2 and gYM_2 's for large groups important.

In [19], the large-N limit of the U(N) YM $_2$ on a sphere was studied. There it was shown that the above mentioned summation is replaced by a (functional) integration over the continuous parameters of the Young tableaux corresponding to the representation. Then the saddle-point approximation singles out a so-called classical representation, which dominates the integration. In this way, it was shown that the free energy of the U(N) YM $_2$ on a sphere with the surface area $A < A_c = \pi^2$ has a logarithmic behavior [19]. In [14], the free energy was calculated for areas $A > \pi^2$, from which it was shown that the YM $_2$ on a sphere has a third-order phase transition at the critical area $A_c = \pi^2$, like the well known Gross-Witten-Wadia phase transition for the lattice two dimensional multicolour gauge theory [20, 21]. The phase structure of the large-N YM $_2$, generalized YM $_2$'s, and nonlocal YM $_2$ on a sphere were further discussed in [15,17,22,23]. It is also seen that for surfaces with no boundaries, only the genus-zero surface (i.e. the sphere) has a nontrivial saddle-point approximation, and all other surfaces have trivial large-N behavior [19].

In [24–26], the expectation value of the Wilson loops corresponding to the large-N U(N)-YM $_2$ on a sphere was investigated. The study was again based on the fact that in the large-N limit, a certain representation of the group is singled out in the partition function and the expectation value of any quantity is essentially the value of that quantity calculated at that representation. Using this, the behavior of the expectation values of the Wilson loops for large-N YM $_2$ was investigated for small and large areas.

In this paper we want to study the expectation-value of the Wilson loops in the fundamental representation, corresponding to large-N YM₂ and gYM₂'s on a sphere. In section 2, the expectation values of a Wilson loop is obtained in terms of the the area enclosed by the loop and the density of the dominant representation. In section 3, the behaviors of the free energy and the expectation values of the Wilson loops for areas near the critical value are investigated by some general arguments. In section 4, we justify explicitly the results of section 3 for two special cases YM₂ and $G(z) = z^4$ gYM₂ models. Finally, in section 5 the large-area behavior of the expectation value of the Wilson loops is studied.

2 The expectation values of the Wilson loops

Following [11,17], a generalized U(N) Yang-Mills theory on a surface is characterized by a function Λ :

$$\Lambda(R) = \sum_{k=1}^{p} \frac{a_k}{N^{k-1}} C_k(R), \tag{1}$$

where R denotes a representation of the group U(N), a_k 's are constants, and C_k 's are the Casimirs of the group defined through

$$C_k = \sum_{i=1}^{N} [(n_i + N - i)^k - (N - i)^k].$$
 (2)

 n_i 's are nonincreasing integers characterizing the representation. It is assumed that p is even and $a_p > 0$. For simplicity, from now on it is further assumed that all a_k 's with odd k's vanish. The partition function of such a theory on an orientable genus g surface with n boundaries is

$$Z_{g,n}(A; U_1, \dots, U_n) = \sum_{R} d_R^{2-2g-n} \chi_R(U_1) \cdots \chi_R(U_n) \exp[-A\Lambda(R)],$$
 (3)

where U_i 's are the holonomies on the boundaries, A is the area of the surface, d_R is the dimension of the representation R, and χ_R is the character of the representation R.

Consider a simple (not self-crossing) loop, which divides a sphere into two regions of areas A_1 and A_2 . The expectation value of the Wilson loop corresponding to this loop is (following [11,17,24])

$$W_r(A_1, A_2) = \frac{1}{Z_0(A_1 + A_2)} \int dU \ Z_{0,1}(A_1, U) \ Z_{0,1}(A_2, U^{-1}) \frac{1}{d_r} \chi_r(U), \quad (4)$$

where r is a representation of U(N), $Z_0(A)$ is the partition function on the sphere, and the integration is done over the group. Eq. (4) can be rewritten as

$$W_r(A_1, A_2) = \frac{1}{d_r Z_0(A_1 + A_2)} \sum_{R,S} d_R d_S \langle R, r | S \rangle \exp[-A_1 \Lambda(R) - A_2 \Lambda(S)],$$

(5)

where the summations run over irreducible representations of U(N), and $\langle R, r | S \rangle$ is the number of representations S in the tensor product of the representations R and r. From now on, the representation r is taken to be the fundamental representation, the dimension of which is N.

For large N, the summation on R is dominated by the so called classical representation, which maximizes the product $d_R \exp[-A_1 \Lambda(R)]$, as was shown in, for example, [17]. To obtain this representation, it is convenient to introduce the new parameters

$$x := \frac{i}{N},$$

$$n(x) := \frac{n_i}{N},$$

$$h(x) := -n(x) - 1 + x,$$
(6)

the density

$$\rho(h) := \frac{\mathrm{d}x}{\mathrm{d}h},\tag{7}$$

and the function

$$G(z) := \sum_{k=0}^{p} a_k (-z)^k.$$
 (8)

Then the density corresponding to the classical representation is characterized by

$$\frac{A}{2}G(z) - \int dy \ \rho(y) \ \ln|z - y| = \text{const.}, \qquad \text{iff } \rho(z) \neq 1 \text{ and } -a \leq z \leq a,$$

$$\int_{-a}^{a} dz \ \rho(z) = 1, \tag{9}$$

where a is a positive number to be determined through the above conditions. Then, using arguments similar to those used in [24], one arrives at

$$W(A_1, A_2) = \int_{-a}^{a} \frac{\mathrm{d}z}{\pi} \sin[\pi \, \rho(z)] \, \exp\left[-P \int_{-a}^{a} \mathrm{d}y \, \frac{\rho(y)}{z - y} + A_2 \, G'(z)\right], \quad (10)$$

where P means Cauchy's principal value, or

$$W(A_1, A_2) = -\oint_C \frac{\mathrm{d}z}{2\pi i} \exp\left[-\int_{-a}^a \mathrm{d}y \, \frac{\rho(y)}{z - y} + A_2 \, G'(z)\right]. \tag{11}$$

G'(z) denotes the derivative of G(z), and C is a counterclockwise contour encircling the real segment [-a,a]. Defining the function H of a complex variable z as

$$H(z) := \int_{-a}^{a} \mathrm{d}y \, \frac{\rho(y)}{z - y},\tag{12}$$

(as it was defined in [17]), one can rewrite (11) as

$$W(A_1, A_2) = -\oint_C \frac{\mathrm{d}z}{2\pi i} \exp\left[-H(z) + A_2 G'(z)\right], \tag{13}$$

It was seen in [17] that H is analytic except on the real segment [-a, a]. In Yang-Mills case where $G(z) = (1/2)z^2$, (13) leads to one derived in [24,25].

3 Free energy and the expectation values of the Wilson loops near the critical area

As it was discussed in [14,17], if the area of the sphere is less than the critical value $A_{\rm c}$, then the density ρ is always less than 1 and the free energy of the system is a smooth function of the area. For areas larger than $A_{\rm c}$, however, ρ becomes equal to 1 on certain segments. This produces a discontinuous behavior in the area-dependence of the free energy, at $A=A_{\rm c}$. There was an argument in [23] to obtain the degree of this nonsmoothness, i.e. the order of this phase transition. However, the argument is problematic and, as will be shown here, leads to incorrect results at some cases.

Let us denote the correct density for $A > A_c$ (the strong region) by ρ_s and the density satisfying the saddle-point equation, but not subject to not exceeding 1, by ρ_w . Define L_w and L_s as the regions where ρ_s is less than one or equal to one, respectively; and L_w' and L_s' as the regions where ρ_w is less than one or greater than one, respectively. The length of L_s is denoted by 2b, so the length of L_s' is of the order 2b. We have

$$\frac{A}{2}G'(z) = P \int dy \, \frac{\rho_s(y)}{z - y}, \quad z \in L_w, \tag{14}$$

$$\frac{A}{2}G'(z) = P \int dy \frac{\rho_w(y)}{z - y}, \quad z \in L'_w \cup L'_s.$$
(15)

where ρ_s (ρ_w) has been defined zero outside $L_w \cup L_s$ ($L'_w \cup L'_s$). Defining

$$\alpha := \max(\rho_w) - 1,\tag{16}$$

and using the ansatz

$$\rho_{s0}(y) := \min[\rho_w(y), 1] \tag{17}$$

as an approximate solution to the eq.(14), it is seen that the difference of the left-hand side with the right-hand side calculated at $\rho_s = \rho_{s0}$, is of the order α when the distance of z from L_s is of the order b, and of the order a when the distance of b from b is large compared to b. To see this, one notes that

$$D(z) := \frac{A}{2} G'(z) - P \int dy \, \frac{\rho_{s0}(y)}{z - y},$$

$$= P \int dy \, \frac{\rho_{w}(y) - \rho_{s0}(y)}{z - y},$$

$$= P \int_{L'} dy \, \frac{\rho_{w}(y) - 1}{z - y}, \quad z \in L'_{w}.$$
(18)

The numerator of the integrand is of the order α , and the length of the integration region is of the order b. If the distance of z from L_s (or L'_s) is of the order b, then the denominator of the integrand is of the order b, and the integral would be of the order α . If the distance of z from L_s (or L'_s) is large compared to b, then the denominator of the integrand is of the order one, and the integral would be of the order αb .

Defining $\delta \rho_s$ through

$$\rho_s =: \rho_{s0} + \delta \rho_s, \tag{19}$$

it is seen that

$$D(z) = P \int dy \frac{\delta \rho_s(y)}{z - y}, \quad z \in L_w.$$
 (20)

 $\delta \rho_s$ is at most of the order α . But $\delta \rho$ cannot be of order α everywhere. In fact, if it is of the order α in a region large compared to b, then the right-hand side would be of the order α for some points z the distance of them from L_s (or L'_s)

is large compared to b. But D(z) is of the order αb when the distance of z from L_s (or L'_s) is large. So,

$$\rho_s(z) - \rho_w(z) = \begin{cases} O(\alpha), & z \in L_s'' \\ o(\alpha), & z \notin L_s'' \end{cases}, \tag{21}$$

where L_s'' is a region around L_s , the length of which is of the order b. Using (21) and

$$H_{s,w}(z) := \int \mathrm{d}y \, \frac{\rho_{s,w}(y)}{z - y},\tag{22}$$

it is seen that

$$H_s(z) - H_w(z) = \int_{L_s} dy \, \frac{1 - \rho_w(y)}{z - y} + \int_{\mathbb{R} \setminus L_s} dy \, \frac{\delta \rho_s(y)}{z - y} = o(\alpha), \quad |z| \gg a, \quad (23)$$

where a is of the order of the length of the region in which the integrand does not vanish.

For large values of z, the difference $H_s(z)-H_w(z)$ is an analytic even function of b [17]. This means that this difference can be expanded like

$$H_s(z) - H_w(z) = c_k(z) b^{2k} + c_{k+1}(z) b^{2k+2} + \cdots,$$
 (24)

where k is a positive integer. If at the point ρ_w attains its maximum, the 2m'th derivative of ρ_w is its first nonvanishing derivative, then b^{2m} is of the order α . As the difference $H_s(z) - H_w(z)$ is $o(\alpha)$, it turns out that k must be greater than m. So one arrives at

$$H_s(z) - H_w(z) \sim b^{2m+2}$$
, for large z (25)

or

$$H_s(z) - H_w(z) \sim \alpha^{1 + (1/m)}, \text{ for large } z.$$
 (26)

The free energy F is defined through

$$F := -\frac{1}{N^2} \ln Z,\tag{27}$$

and one has [17]

$$F'(A) = \int_{-a}^{a} dy \, \rho(y) \, G(y). \tag{28}$$

Using (12), one can rewrite this as

$$F'(A) = \oint_{G_{-z}} \frac{\mathrm{d}z}{2\pi i} H(z) G(z), \tag{29}$$

where C_{∞} is a large counterclockwise circle. Now, using (26), it is seen that

$$F_s'(A) - F_w'(A) \sim (A - A_c)^{(2l-1)[1+(1/m)]},$$
 (30)

and from that

$$F_s(A) - F_w(A) \sim (A - A_c)^{1 + (2l - 1)[1 + (1/m)]},$$
 (31)

where the (2l-1)'th derivative of α with respect to A is the first nonvanishing derivative of α with respect to A, so $\alpha(A) \sim (A - A_c)^{2l-1}$. It is seen that in general the last equation differs that found in [23]. The two, however, coincide if l=1. The flaw in the argument of [23] is that from the fact that the integral of the difference $\rho_s - \rho_w$ on L_w is of order αb , it was deduced that $\rho_s - \rho_w$ itself is of order αb , which is not true, as seen from (21). However, for typical theories where l=m=1, it is still true that the system exhibits a third order phase-transition at $A=A_c$.

Using (13) and (26), one can also investigate the nonsmoothness behavior of the expectation values of the Wilson loops. Using (13), It is seen that

$$W_s - W_w \sim (A - A_c)^{(2l-1)[1+(1/m)]},$$
 (32)

so that for typical theories, there is a second order discontinuity in the expectation values of the Wilson loops. For example for all gYM₂ theories with $G(z) = a_k z^{2k} (k \ge 1)$, including YM₂, it has been shown that [23]

$$\alpha(A) = \left(\frac{A}{A_{\rm c}}\right)^{\frac{1}{2k}} - 1. \tag{33}$$

This shows that the derivative of α with respect to A never vanishes, or l=1. Also the second derivative of ρ_w is negative at the maximum of ρ_w , which gives m=1. So for all theories with $G(z)=a_k\,z^{2k}$,

$$W_s - W_w \sim (A - A_c)^2. \tag{34}$$

4 Explicit evaluation of the order of discontinuity of Wilson loops for YM_2 and the gYM_2 with $G(z)=z^4$

From (12), and using the fact that ρ is even if G is even, one has

$$H(z) = \frac{1}{z} + \frac{1}{z^3} \int_{-a}^{a} dy \ y^2 \rho(y) + \frac{1}{z^5} \int_{-a}^{a} dy \ y^4 \rho(y) + \cdots$$
 (35)

Using this and (13), one arrives at

$$W_{s} - W_{w} = \oint_{C} \frac{\mathrm{d}z}{2\pi i} \exp\left[-H_{w}(z) + A_{2} G'(z)\right] \left[H_{s}(z) - H_{w}(z) + \cdots\right],$$

$$= \oint_{C} \frac{\mathrm{d}z}{2\pi i} \exp\left[-H_{w}(z) + A_{2} G'(z)\right] \left[\frac{\Delta(y^{2})}{z^{3}} + \frac{\Delta(y^{4})}{z^{5}} + \cdots\right], \quad (36)$$

where

$$\Delta(y^n) := \langle y^n \rangle_s - \langle y^n \rangle_w,$$

$$= \int dy \ y^n \rho(y) \left[\rho_s(y) - \rho_w(y) \right]. \tag{37}$$

It is seen that $W_s - W_w$ is proportional to $\Delta(y^2)$. For small values of A_2 , one can expand $\exp[A\,G'(z)]$ and it is seen that only a finite number of terms in (36) are needed to obtain the behavior of $W_s - W_w$. (In the product of $[G'(z)]^n [H_s(z) - H_w(z)]$, one needs to keep only terms of z^k for $k \ge -1$.)

For example, if $G(z) = (1/2) z^2$, using the first two terms one can obtain the behavior of $W_s - W_w$ up to order A_2^4 . For this theory, one has (using [14])

$$\Delta(y^2) = 2(F'_s - F'_w),$$

$$= \frac{2}{\pi^2} \left(\frac{A - A_c}{A_c}\right)^2 + \cdots,$$
(38)

and

$$\Delta(y^4) = \frac{4}{\pi^2} \left(\frac{A - A_c}{A_c}\right)^2 + \cdots,$$
 (39)

which shows that leading term of $W_s - W_w$ is proportional to $(A - A_c)^2$, at least up to order $(A_2)^4$.

As a second example, one can take $G(z) = z^4$. Following [17], one can expand $H_s - H_w$ near the critical area, and expand it in terms of the powers of z^{-1} , from which $\Delta(y^2)$ (the coefficient of z^{-3}) is calculated to be

$$\Delta(y^2) = \frac{551}{486\pi^2} \left(\frac{A - A_c}{A_c}\right)^2 + \cdots, \tag{40}$$

which shows that the leading term of $W_s - W_w$ is proportional to $(A - A_c)^2$, at least up to first order in A_2 .

5 The expectation values of Wilson loops for large areas

To study the Wilson loop on the plane $(A \to \infty)$, one notes that for large values of A, the first equation of the set (9) cannot be fulfilled with finite ρ . This shows that as A tends to ∞ , the density ρ becomes equal to one everywhere in (-a, a). Then the second equation shows that a tends to (1/2) as A tends to ∞ :

$$\lim_{A \to \infty} \rho(z) = 1, \qquad -a < z < a,$$

$$\lim_{A \to \infty} a = \frac{1}{2}.$$
(41)

This shows that

$$\lim_{A \to \infty} H(z) = \ln \frac{z + \frac{1}{2}}{z - \frac{1}{2}}.$$
 (42)

For A large but not infinite, one can use (12) to expand H(z) around the value of H for infinite A. The result would be

$$H(z) = \ln \frac{z + \frac{1}{2}}{z - \frac{1}{2}} + \sum_{i,n} \frac{\alpha_{i,n}(A)}{(z - z_i)^n},$$
(43)

where n runs over positive integers and z_i 's are points in $\left[-\frac{1}{2}, \frac{1}{2}\right]$ around them ρ differs from one (for large but not infinite areas). Among these points are $-\frac{1}{2}$ and $\frac{1}{2}$. $\alpha_{i,n}$'s should of course tend to zero as A tends to infinity. Using the above expansion in (13), one arrives at

$$W(A_1, A_2) = \exp\left[A_2 G'\left(-\frac{1}{2}\right)\right] + \sum_i \beta_i(A_2, A) \exp[A_2 G'(z_i)], \tag{44}$$

where β_i 's are sums of polynomials in A_2 times functions of A. One can write the above equation in a form more manifestly-symmetric with respect to A_1 and A_2 :

$$W(A_1, A_2) = \exp\left[A_2 G'\left(-\frac{1}{2}\right)\right] + \exp\left[A_1 G'\left(-\frac{1}{2}\right)\right] + \sum_i \tilde{\beta}_i(A_2, A) \exp[A_2 G'(z_i)], \tag{45}$$

where

$$\tilde{\beta}_i := \begin{cases} \beta_i, & z_i \neq \frac{1}{2} \\ \beta_i - \exp\left[A G'\left(-\frac{1}{2}\right)\right], & z_i = \frac{1}{2} \end{cases}$$

$$\tag{46}$$

and use has been made of the fact that G' is an odd function.

For YM₂, one can perform a more explicit calculation. Here it is known that for $A > A_{\rm c}$, the density is equal to one for -b < z < b, where b is some positive number less than a. Also, it is seen that a and b both tend to $\frac{1}{2}$ as A tends to ∞ . So there are only two z_i 's and one has

$$H(z) = \ln \frac{z + \frac{1}{2}}{z - \frac{1}{2}} + \alpha(A) \left[\frac{1}{(z - \frac{1}{2})^2} - \frac{1}{(z + \frac{1}{2})^2} \right] + O(\alpha^2), \tag{47}$$

where

$$\alpha = \frac{1}{2} \left(\frac{1}{2} - b \right)^2 + \int_b^a dy \left(y - \frac{1}{2} \right) \rho(y).$$
 (48)

(One can see that H is an odd function and its residue at z=0 should be equal to one, so there are no first-order poles at $z=\pm\frac{1}{2}$.) Putting this in (13), one

arrives at

$$W = -\oint \frac{\mathrm{d}z}{2\pi i} \frac{z - \frac{1}{2}}{z + \frac{1}{2}} \left\{ 1 - \alpha \left[\frac{1}{(z - \frac{1}{2})^2} - \frac{1}{(z + \frac{1}{2})^2} \right] + O(\alpha^2) \right\} \exp(A_2 z),$$

$$= \exp\left(-\frac{A_2}{2} \right) \left[1 + \alpha \left(-1 - A_2 + \frac{A_2^2}{2} \right) \right] + \alpha \exp\left(\frac{A_2}{2} \right) + O(\alpha^2). \tag{49}$$

It is known that for $A_2=A,\,W_s$ should become unity. This shows that up to the leading order,

$$\alpha = \exp\left(-\frac{A}{2}\right). \tag{50}$$

So,

$$W = \exp\left(-\frac{A_2}{2}\right) \left[1 + \exp\left(-\frac{A}{2}\right) \left(-1 - A_2 + \frac{A_2^2}{2}\right)\right] + \exp\left(-\frac{A_1}{2}\right) + \cdots$$
(51)

This is the same relation obtained in [24], using a different method.

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